XXVII. *On the reduction to a vacuum of* Captain KATER's *convertible pendulum.* By Captain EDWARD SABINE, *of the* Royal Artillery, *Secretary to the Royal Society.*

THE convertible pendulum with which Captain KATER made his celebrated and very admirable experiments, on the length of the pendulum vibrating seconds in vacuo in Portland Place, was deposited, after the completion of those experiments, in the cabinet of the Royal Society.

The experiments of Captain KATER were made, as is well known, in the free air of the ordinary atmosphere; and the influence of the air in retarding the vibrations, and thereby interfering with the simple effect of the earth's attraction on the pendulum, was computed and allowed for on a principle universally received by mathematicians and experimentalists at that period.

It has been shown by recent investigations, theoretical and experimental, that the principle on which the reduction to a vacuum was then computed is erroneous: and it is a consequence of those investigations, that further experiments are necessary with the convertible pendulum employed by Captain KATER, in order that the true vibration in a vacuum, corresponding to the distance between its knife edges, may be known; and that the more correct length of the seconds pendulum, such as Captain KATER would himself have determined it had he been aware in 1817 of what has subsequently been discovered, may be substituted for the result published by him in the Philosophical Transactions for 1818.

The apparatus, of which an account has been presented to the Society in the present session, in which pendulums can be vibrated both in air of ordinary density and in a highly rarefied medium approaching to a vacuum, affords the means of making these further experiments. At the wish of Captain KATER, and in compliance with a request of the council of the Royal Society, I have undertaken to make them; and hope, at the commencement of the next session,

to present to the Society the result which shall be obtained, as the result of Captain KATER's pendulum.

From the principles developed in the recent investigation into the action of the air on the vibrations of a pendulum, it was to be inferred that a convertible pendulum, such as the one Captain KATER employed, would in two respects be affected by the medium in a different manner from that which he had supposed: namely, first, in respect to its presumed convertibility; for, since the amount of the retardation occasioned by the air is dependent in part on the external figure of the body vibrating, and as the two ends of the pendulum are not symmetrical, the one being furnished with a large weight, and the other with a much smaller weight and of a different form, the reduction to a vacuum will not be of the same amount when the pendulum is suspended with the great weight uppermost, as when suspended with the great weight below; and consequently the pendulum is erroneously supposed to be convertible when the vibrations in air are identical. And second, in respect to the amount of the retardation produced by the air, which would be considerably greater than the quantity computed on the simple consideration of buoyancy.

The experiments that have already been made with this pendulum in the vacuum apparatus, both in the state in which Captain KATER constructed and employed the pendulum, and with certain alterations which I have found it expedient to make in its tail pieces, have fully confirmed these inferences; and in the opinion of those whose judgement I have reason to respect, possess an interest in the elucidation, and further experimental illustration of the mode in which a medium acts on the pendulum in retarding its vibration, which makes it desirable that I should communicate the present brief account of them to the Society before the recess.

The pendulum having been conveyed to Greenwich, was examined and found in excellent order; the knife edges were as clean and apparently as perfect as when first used; the smaller weight was well secured by its screws, and the slider was at 18.6 divisions towards the greater weight: the rate of vibration on each of the knife edges was then tried by a few coincidences, and found so nearly identical, as to make it probable that little or no alteration had been made in the positions of the weight and slider since Captain KATER's experiments: and as the reduction to a vacuum for each position of the pendulum,

which was the present object of inquiry, did not require that the convertibility should be very rigorously established, the following experiments were made to determine the comparative rate, in air of ordinary density, and in highly rarefied air; first with the great weight below; and second, with the great weight uppermost.

Great weight below. April 21, 1829. Clock by Dent making 86466.00 Vibrations.

Observers.	No. of Coincid.	Standard Therm.	Barom. or Gauge.	Times of Disapp.	Times of Re-app.	Coincidence.	Arc registered and true Arc.	Mean Interval.	Correction for Arc.	Reduction to 49°.	Corrected Vibrations at 49°.
				m s	m s		Div.				
Captain Sabine.	1	....	....	34 20	34 33	} h m s 2 41 38.17	0.68 = 0.82				
	2	50.0	29.630	41 31	41 45						
	3	....	....	48 42	48 58			432.97	+0.36	+0.24	86067.18
	17	....	....	29 35	30 12						
Captain Sabine.	18	....	....	36 46	37 25	} 4 44 18.6	0.15 = 0.18				
	19	49.1	29.630	43 58	44 38						
	20	....	....	51 11	51 52						
	21	....	....	58 23	59 06						
		49.55	29.630; Capill. + 0.019; Reduction to 32° −0.053; = 29.596.								86067.18

April 22. Clock making 86466.00 Vibrations.

Observers.	No. of Coincid.	Standard Therm.	Barom. or Gauge.	Times of Disapp.	Times of Re-app.	Coincidence.	Arc registered and true Arc.	Mean Interval.	Correction for Arc.	Reduction to 49°.	Corrected Vibrations at 49°.
				m s	m s		Div.				
Mr. Taylor.	1	....	....	19 49	19 59	} h m s 0 27 23	0.92 = 1.10				
	2	47.7	1.05	27 18	27 28						
	3	....	....	34 47	34 57			449.44	+1.60	−0.24	86082.60
Mr. Taylor.	13	....	....	49 39	49 53	} 1 57 16.33	0.73 = 0.88				
	14	47.7	1.05	57 10	57 24						
	15	....	....	04 39	04 53						
		47.7; + Index correction for a rarefied medium 0.75; = 48.45. Gauge 1.05.									86082.60

Observers.	No. of Coincid.	Standard Therm.	Barom. or Gauge.	Times of Disapp.	Times of Re-app.	Coincidence.	Arc registered and true Arc.	Mean Interval.	Correction for Arc.	Reduction to 49°.	Corrected Vibrations at 49°.
				m s	m s		Div.				
Mr. Taylor.	1	....	....	27 19	27 32	} h m s 2 34 39.33	0.79 = 0.95				
	2	49.0	29.350	34 33	34 46						
	3	....	....	41 46	42 00			434.28	+0.65	−0.17	86068.26
Captain Sabine.	11	....	....	39 34	39 58	} 3 47 02.17	0.30 = 0.36				
	12	48.2	29.350	46 49	47 15						
	13	....	....	54 07	54 30						
		48.6	29.350; Capill. + 0.019; Reduction to 32° − 0.052; = 29.317.								86068.26

Great weight uppermost. April 23rd. Clock making 86466.00 vibrations.

Observers.	No. of Coincid.	Standard Therm.	Barom. or Gauge.	Times of			Arc registered and true Arc.	Mean Interval.	Correction for Arc.	Reduction to 49°.	Corrected Vibrations at 49°.
				Disapp.	Re-app.	Coincidence.					
				m s	m s						
Captain SABINE. {	1	....	....	55 27	55 39	} h m s 1 02 49.17	Div. 0.60 = 0.72				
	2	49.5	29.604	02 42	02 55			} 438.484	+0.29	+0.26	86072.15
	3	....	....	09 59	10 13						
Captain SABINE. {	12	....	....	15 39	16 09	} 2 23 12.5	0.15 = 0.18				
	13	49.7	29.612	22 55	23 30						
	14	....	....	30 13	30 49						
		49.6	29.608; Capill. + 0.019; Reduction to 32° − 0.052; = 29.575.								86072.15
				m s	m s						
Captain SABINE. {	1	....	....	18 22	18 31	} h m s 3 26 00	Div. 1.20 = 1.44				
	2	49.6	0.90	25 56	26 04			} 454.91	+1.04	+0.39	86087.27
	3	....	....	33 29	33 38						
Mr. TAYLOR. {	34	....	....	28 21	28 52	} 7 36 12	0.25 = 0.30				
	35	48.7	3.30	35 58	36 26						
	36	....	....	43 33	44 02						
		49.15; + Index correction for a rarefied medium 0.75 = 49.9.								Gauge 2.10	86087.27
				m s	m s						
Mr. TAYLOR. {	1	....	....	6 17	6 33	} h m s 8 13 42	Div. 0.98 = 1.176				
	2	49.6	29.640	13 34	13 51			} 439.1	+0.62	+0.04	86072.84
	3	....	....	20 51	21 06						
Mr. TAYLOR. {	13	....	....	33 59	34 24	} 9 41 31.17	0.17 = 0.20				
	14	48.6	29.630	41 18	41 44						
	15	....	....	48 38	49 04						
		49.1	29.635; Capill. + 0.019; Reduction to 32° − 0.052; = 29.602.								86072.84

The vibrations in these experiments were as follows.

With the great weight below :

April 21. Previous to the vibration in the rarefied medium . . . . } Vibrations. 86067.18   Barom. 29.596

April 22. Subsequent to the vibration in the rarefied medium . . . . } 86068.26   Barom. 29.317

Mean . . . . . . 86067.72   Barom. 29.456

The vibrations in a rarefied medium reduced to the same temperature as those in air of ordinary density . . . . } 86082.60   Gauge 1.05

Difference 14.88   28.406

Whence there appears, as the result of the experiment with the great weight below, a difference of 14.88 vibrations per diem, corresponding to a difference of atmospheric pressure of 28.406 inches of mercury at 32°; the temperature of the air of full pressure being 49.07, and that of the rarefied medium 48.45.

And with the great weight uppermost:

		Vibrations.	
April 23. Previous to the vibration in the rarefied medium	}	86072.15	Barom. 29.575
April 23. Subsequent to the vibration in the rarefied medium	}	86072.84	Barom. 29.602
Mean		86072.49	Barom. 29.588
The vibrations in a rarefied medium reduced to the same temperature as those in air of ordinary density	}	86087.27	Gauge 2.10
Difference . . .		14.78	27.488

Whence, with the great weight uppermost, there is found a difference of 14.78 vibrations per diem, corresponding to a difference of atmospheric pressure of 27.488 inches of mercury at 32°; the temperature of the air of full pressure being 49.35, and that of the rarefied medium 49.9.

From these results we obtain 15.7 vibrations per diem as the reduction to a vacuum for the convertible pendulum, as it was used by Captain KATER, vibrating with the great weight below, in air of 49°, under a pressure of 30 inches of mercury at 32°; and 16.1 vibrations per diem, when inverted, or with the great weight uppermost, in air of like temperature and density.

According to the formula by which Captain KATER reduced the vibrations of this pendulum in air to the supposed vibrations in a vacuum, the reduction for each position of the pendulum would be the same, and the amount for air of the named temperature and density would have a little exceeded 7 vibrations per diem.

In the account of Captain KATER's experiments (Phil. Trans. 1818, page 75) it is remarked that on a sudden and considerable change having taken place in the hygrometric state of the atmosphere, vibrations which had been previously identical in each position of the pendulum ceased to be so; an effect which he attributed to an alteration in the weight of the wooden extremities of the pen-

dulum by their loss of moisture as the weather became more dry. As we now know that the general effect of the air in retarding the vibration is more than twice as great as was then imagined, and that the wooden tail pieces, in consequence of their position, have a far greater proportionate influence on the retardation than would be simply due to the diminution which they occasion in the general specific gravity of the pendulum, we perceive increased reason to agree with Captain KATER, and to apprehend so much danger of derangement from this cause, as to make it desirable to avoid altogether the employment of a material susceptible of changes from moisture.

For this reason I determined to substitute tail pieces of brass, with such an alteration in the position or size of the smaller weight as should re-establish the equality of vibration. Previously, however, to this being done, and for the purpose of illustrating more strongly the effect of the wooden tail pieces, I had them reduced to less than half their original length, 10.6 inches being taken off from each, leaving them 6.4 inches from the extremity of the brass bar. By substituting a still smaller weight (of 1925 grains) of the same metal and form, for the smallest weight used by Captain KATER (of 3325 grains), and securing it very nearly in the same position, the pendulum was again rendered nearly convertible, and experiments were made with it in the same succession as before, of which the following are the results.

With the great weight below, there was found a difference of 11.9 vibrations per diem, corresponding to a difference of atmospheric pressure of 28.741 inches of mercury at 32°, the temperature of the air of full pressure being 53.2, and of the rarefied air 53.8.

And with the great weight uppermost a difference of 14 vibrations per diem, corresponding to a difference of 28.15 inches of mercury at 32°, the temperature of the air of full pressure being 51.4, and of the rarefied air 53.8.

Whence we obtain 12.4 vibrations per diem as the reduction to a vacuum, when the wooden tail pieces were shortened from 17 inches to 6.4 inches, and the great weight was below, in air of 53°.5, under a pressure of 30 inches of mercury at 32°; and 14.9 vibrations per diem with the great weight uppermost, in air of the same temperature and density.

The wooden tail pieces were then altogether removed, and slips of brass substituted, extending 7 inches from each extremity of the bar. The pendulum

was rendered convertible in air by the small weight used by Captain KATER, screwed to the bar at a somewhat greater distance from its knife edge than in his experiments; and coincidences being observed in the usual routine, it appeared that with the great weight below, a difference took place of 11.3 vibrations per diem, corresponding to a difference of atmospheric pressure of 28.46 inches of air at 32°, the temperature of the air of full pressure being 58°, and of the rarefied air 55°.5 : and with the great weight uppermost, a difference of 12 vibrations per diem, corresponding to a difference of atmospheric pressure of 28.04 inches of air at 32°, the temperature of the air of full pressure being 60°, and of the rarefied air 60°.5.

Whence we obtain when the great weight is below, 11.8 vibrations per diem as the reduction to a vacuum, in air of 60°, under a pressure of 30 inches of mercury at 32°; and 12.8 vibrations per diem when the great weight is uppermost.

Finally, recapitulating the results obtained with the different modifications of the tail pieces, we have

1. With the wooden tail pieces 17 inches in length . . . . } Great weight above 16.1 Vibrations.; below 15.7 Vibrations.

2. With the wooden tail pieces 6.4 inches in length . . . . } ——————————— 14.9; —— 12.4

3. With the brass tail pieces 7 inches in length . . . . . . } ——————————— 12.8; —— 11.8

In comparing Nos. 1 and 2 with each other, and both with No. 3, we perceive the very great effect which the employment of so light a material as wood for the tail pieces of the pendulum produces, in increasing the difference between the vibrations in air and those in a vacuum. When slips of brass were substituted for the slips of deal employed by Captain KATER, the retardation caused by the air was diminished not less than between 3 and 4 vibrations per diem on the one knife edge, and upwards of 4 vibrations per diem on the other. When the wooden tail pieces were reduced to the same length, or nearly so, as those of brass, the retardation with them was still found greater than with the brass tail pieces of similar size and figure, by 2 vibrations per diem on the one knife edge, and 1 vibration per diem on the other. The retardations were in all cases much more considerable than would have been

computed on the simple consideration of buoyancy: they were particularly so in those instances in which the extremities of the pendulum were of the lighter material; for the increase in the retardation in those instances much exceeded the proportion due to the diminution of the general specific gravity of the pendulum, occasioned by the addition of the small portions of wood.

In viewing the comparative retardations in the two positions of the pendulum, in each of these experiments, we find a confirmation of the inference, noticed in the earlier part of this paper, that in consequence of the want of symmetry in the two ends, the reduction to a vacuum ought not to be the same in the two positions, of the pendulum. We have also a curious exemplification of the influence of the addition of equal portions of matter at each extremity of the pendulum, in diminishing the difference in the retardation occasioned by the disparity in the form and size of the weights. With the short wooden tail pieces, the difference (which with no tail pieces at all would probably have exceeded 3 vibrations per diem) amounted to 2.5 vibrations. With the brass tail pieces it was lessened to 1 vibration. And with the wooden tail pieces of their original length, the effect of the inequality of the weights was almost altogether counterbalanced, the retardation being within half a vibration the same in each position of the pendulum.

Finally, it is curious to perceive how much the result obtained by Captain KATER, as the length of the seconds pendulum, depended on the mere accidental circumstance of the addition of tail pieces to his experimental pendulum: had the circumstances of the experiment been varied in regard to the tail pieces; had they been of brass for example;—or being of wood, had they been of any other length than that which was determined by the accidental circumstance of the relative heights of the clock, and pendulum support;—or had they been altogether omitted and the coincidences observed by means of the bar itself,— a widely different result would in each of these cases have been arrived at.